Concept Paper for Fostering Pure and Applied Mathematics in Sub-Sahara Africa

Name of the project: Research and Graduate Studies in Mathematics and its Applications (RGSMA): A Network Approach

Main aim of the project: The aim of the project is, in general, to promote Mathematics Education in Sub-Sahara Africa, in particular, to:

1. train candidates in the sub-Sahara region at MSc and PhD levels in areas of Mathematics, and
2. conduct collaborative research in Mathematical Modelling, Mathematical Analysis and Geometry.

Location of the project: Our project is hosted by the University of Botswana, Department of Mathematics, with training nodes at the universities of Stellenbosch (South Africa), Namibia, Kwazulu Natal (South Africa), Addis Ababa (Ethiopia), UNISA (South Africa) and NUST (Zimbabwe).

Target Group: The project provides a platform for experienced teachers and researchers in the sub-Sahara African region to coordinate their efforts in training MSc and PhD candidates in Applied and Pure Mathematics.

Currently funding: Our project is supported by the Simmons Foundation. The Foundation provides $80,000.00 per year for 5 years (from Jan 2013-Dec 2017) which will be extended for some more years based upon our achievement. In addition we have received EURO 8,000.00 from the International Mathematical Union (IMU)- CDC Project Grant Program 2015 to extend our coverage to other areas of the region not previously covered.

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Contact Details of Initiators of the Network

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3. Prof. Fortuné Massamba, Department of Mathematics, University of Kwazulu Natal, email: Massamba@ukzn.ac.za
4. Dr. Habtu Zegeye, Department of Mathematics, University of Botswana. Pvt. Bag, 00704, email: habtuzh@yahoo.com

Brief description: Sub-Sahara Africa faces many challenges among them

1. Economic poverty, knowledge poverty, human diseases such as HIV/AIDS, Malaria, diarrhoea, plant and animal diseases. Malaria and diarrhoea are responsible for over 50% of children deaths under the age of five, while plant diseases compounded by frequent droughts have resulted in low crop yields and consequently poor nutrition for many residents of sub-Sahara Africa.

2. Sub-Sahara Africa Universities lack skilled human resource to train candidates at Msc and Ph.D levels who can teach at higher institutions and conduct research on the existing problems of Africa. Most of the staff members of sub-Sahara African universities are master's degree holders but they are forced to teach courses at master's level. The staffing situation will continue to be bad for the foreseeable future, a situation which has resulted in teaching a narrow range of courses.

To tackle the challenges mentioned above, a group of universities in Southern Africa, under the auspice of the Southern Africa Mathematical Sciences Association (SAMSA) are working hard to address the shortage of skilled human resource and low research output in the Mathematical Sciences through a regional network approach which optimizes the sharing and use of available skilled human and physical resources to sustain the limited training programs.

The first training program in Southern Africa was a joint initiative between SAMSA and NUFU, a Norwegian funding agency. This regional program in Mathematical modelling ran from 1996 to 2004 and produced 150 MSC graduates and 25 PhD graduates, all of whom are professors in the various universities of Southern Africa.

The current program, ''Research and Graduate Studies in Mathematics and its Applications: A Network Approach", centred at the University of Botswana, is an initiative by the Sab-Saharan researchers listed above and is designed to train students in many Sub-Saharan Africa Universities at MSC and PhD levels in order to increase the pool of skilled human resource and reduce the brain drain. We believe that the candidates should be trained within the region where their skills are already scarce. The upgrade project would have the effect of improving the quality and effectiveness of teaching and learning, and research. Our project focuses on areas of Bio-Mathematics, supported by Analysis and Geometry:

1.1 Bio-Mathematics Modelling Prof. Edward M LUNGU:

The immerging and re-emerging human diseases have caused immense suffering to humans in sub-Sahara Africa. To tackle this, we have to motivate new approaches in mathematical biology that utilize the study of the spaces of knots on a compact connected manifold, an area which deals with algebraic and differential topological properties of spaces of paths and loops in manifolds. Because Southern Africa is an open region allowing free movement of humans between countries, the use of deterministic theory to determine prevalence and incidence is questionable. We intend to apply advanced theories in stochastic analysis to estimate the rates of incidence and prevalence and to determine stability of the equilibrium points.

In addition, although sub-Sahara Africa has vast areas of fertile land, the sub-region suffers from shortages of food because the sub-region lacks the technology to engineer foods and the resources to control the pests. There is very little research being conducted in this area. Indeed there are no, or very few, models developed in this area for the sub-SaharaAfrican region. For the sake of food security for the region and the well being of the inhabitants, we intend to initiate research in this important area. The research will include optimization for food collection, fertilizer distribution, etc. Furthermore, we want to develop harvesting models that will provide answers to sustainability of wildlife and domestic livestock, two vital industries to our economies.

We have assembled a team of bio-mathematicians from departments of mathematics in the region. The team consists of:

1. Prof. E. M. Lungu, University of Botswana
2. Prof. F Gideon, University of Namibia
3. Dr. M. Kgosimore, Botswana College of Agriculture
4. Prof. F. Nyabadza, University of Stellenbosch, South Africa
5. Prof. S. D. Musekwa, NUST, Zimbabwe
6. Mr. B. Lephodisa, University of Botswana

Publications in the area of Bio mathematics/Financial Mathematics related to the RGSMA project:

1. **Barbara Szomolay and Edward M. Lungu.** A Mathematical Model for the Treatment of AIDS-Related Kaposi's Sarcoma. Journal of Biological Systems. (Accepted and to appear 2014).
2. **E.Lungu,T. J. Massara, E. Ndelwa, N. Ainea, S. Chibaya and N. J. Malunguza (2013).** Mathematical Modeling of the HIV/Kaposi's Sarcoma Co-infection Dynamics in Areas of High HIV Prevalence. Computational and Mathematical Methods in Medicine. Volume 2013, Article ID 753424, Pg 1-12.
3. **Avner Friedman and Edward M. Lungu** (2013). Can Malaria parasite Pathogenesis be Prevented by Treatment with Tumor Necrosis Factor-Alpha? Mathematical Biosciences and Engineering. Volume 10, Number 3, Pg 1-17.

4. **FaraimunasheChirove and Edward M. Lungu**(2013). Effects of Replicative Fitness on Competing HIV Strains. Bio Systems. Volume 113, 28-36.
5. **Obonye Doctor, Elias Offen and Edward M.Lungu.**Solution to a PortfolioSelection Problem Under Different Stock Processes With Stochastic Income. Presented at a conference held at the University of Stellenbosch, South Africa in November 2013 (Manuscript to be submitted for publication).
6. **E. Koga and E. M. Lungu.** A Mathematical Model for Vertical Transmission of Malaria. Presented at a conference held at the University of Stellenbosch, South Africa in November 2013 (Manuscript to be submitted for publication).
7. **Kerabeng Moalosi and Edward Lungu.** An optimal Investment problem for an investor subject to undiversifiable income risk. Results to be presented at ICM 2014 in Seoul, Korea in August 2014.
8. **Luiz H. R. Alvarez, Edward Lungu and BerntOksendal.** Optimal Multi-Dimensional Stochastic Harvesting with Density-Dependent Prices. Afrika Matematika, Vol. 2015.
9. **Elias Offen and Edward Lungu.** Pricing a European Option in a Black-Scholes Quanto Market When Stock Price is a Semi-martingale. Journal of Mathematical Finance (Accepted April 2015)
10. **Bwalya Lungu and Edward Lungu.** Impact of beliefs on the spread of Ebola. CAS, Second conference on stochastics of environmental and financial economics. University of Oslo, Norway April 2015.
11. **Edward Lungu.** Transplacental Transmission of Plasmodium Malaria in Pregnant Mothers. The 31st SAMSA conference held at Victoria Falls, Zimbabwe from 21st to 29th November 2014.
12. **Obonye Doctor and Edward Lungu.** An Asymptotic Analysis for Optimal Portfolio with Proportional Transaction Costs: A Foreign Exchange Case. The 31st SAMSA conference held at Victoria Falls, Zimbabwe from 21st to 29th November 2014.
13. **Kerabeng Moalosi and Edward Lungu.** The impact of multi-company stock management on the evaluation of executive remuneration The impact of multi-company stock management on the evaluation of executive remuneration. The 31st SAMSA conference held at Victoria Falls, Zimbabwe from 21st to 29th November 2014.
14. **Enock Koga and Edward Lungu.** Congenital Malaria Infection. The 31st SAMSA conference held at Victoria Falls, Zimbabwe from 21st to 29th November 2014.

1.2 Pure Mathematics Prof. Habtu Zegeye Prof. J-B Gatsinzi & Prof. Massamba

Pure Mathematics is a building block of Mathematics. The strength of Mathematics and the variety of its applications will depend on the strength of its building blocks. The initiators of this project recognize the importance of Mathematics to the building of a stronger Science

and Technology culture and would like to see bold steps taken to promote Pure Mathematics in Africa. This project, therefore, propose a network in the Africa region offering an MSC and PhD degrees in Pure Mathematics (in particular, Analysis, Algebra and Geometry).

The degree programme is aimed at giving quality courses of Pure Mathematics that are essential for a good understanding of Applied Mathematics. Many funders neglect this important building block for understanding mathematics and opt for mathematical modelling and applications of mathematics programmes.

1.2.1 Analysis Prof. Habtu Zegeye : The analysis group focuses on latest developments in Nonlinear Operator Theory and its Applications. Nonlinear Operator Theory falls within the general area of Nonlinear Functional Analysis, an area which has been of increasing research interest in recent years. Nonlinear Operator Theory applies to diverse nonlinear problems in many areas Applied Mathematics such as differential equations, nonlinear ergodic theory, game theory, optimization problems, control theory, variational inequality problems, Equilibrium Problems, split feasibility problems and countless others. This issue will reflect both the state-of-the-art theoretical research and important recent advances in applications. We are interested in high quality articles that will outline recent progress in this area of research. This will strengthen the quality and quantity of science in Africa.

We have assembled a team of mathematicians in analysis. The team consists of:
1. Prof. H. Zegeye/co-ordinator/ University of Botswana, Botswana
2. Prof. N. Shahazad, King Abdul Aziz university, Saudi Arabia
3. Dr. M. G. Sangago, Addis Ababa university, Ethiopia
4. Dr. E. U. Ofoedu, University of Nigeria, Nigeria
5. Prof. M. Robdera, University of Botswana, Botswan

Publications in the area of Analysis related to the RGSMA project:

1. **H. Zegeye and N. Shahzad** (2013): Approximation Analysis for a common fixed point of finite family of mappings which are asymptotically k-strict pseudocontractive in the intermediate sense, Journal of Applied Mathematics, Volume 2013(2013), Article ID 831737, 7 pages.
2. **HabtuZegeye** (2013); Approximation Methods for a Fixed Point of Asymptotically Pseudocontractive Mappings in the Intermediate Sense, JP Journal of Fixed Point Theory and Applications, Volume 8, Issue 1, Pages 49 – 71.
3. **H. Zegeye and N. Shahzad** (2014), Strong convergence theorems for a solution of a finite family of Bregman weak relatively nonexpansive mappings in reflexive Banach spaces, The Scientific World Journal, Volume 2014 (2014), Article ID 493450, 8 pages.

4. **Naseer Shahzad, ArifRafiq and HabtuZegeye**(2014), Implicit approximation\ scheme for\ the of k-positive definite operator equation, Abstract and Applied Analysis, Volume 2014, Article ID 683295, 6 pages.

5. **Abebe R. Tufa** andHabtuZegeye: An algorithm for finding a common point of the solutions of fixed point and variational inequality problems in Banach spaces, *Arabian Journal of Mathematics*, DOI: 10.1007/s40065-015-0130-0.

6. **Abebe R. Tufa** andHabtuZegeye: Iterative Solutions of Nonlinear Integral Equations of Hammerstein Type*; Journal of Fixed Point Theory and Applications*, (submitted).

7. SebsibeTeferiWoldeamanuel and Mengistu Goa Sangago and HabtuZegeye, {Strong Convergence Theorems for a Common Fixed Point of a Finite Family of Lipschitz Hemicontractive Multivalued Mappings, *Adv. Fixed Point Theory,* 5 (2015), No. 2, 228-253, ISSN: 1927-6303.

8. SebsibeTeferiWoldeamanuel and Mengistu Goa Sangago and HabtuZegeye, A one-step Implicit Iterative Process for Common Fixed Points of Two Asymptotically Nonexpansive Mappings in Certain Banach Spaces, Numerical Functional Analysis and Optimization, (submitted).
9. H. Zegeye and N. Shahzad (2014); An algorithm for finding a common point of the solution set of a variational inequality and the fixed point set of a Bregman relatively nonexpansive mapping, *Appl. Math. Comput*, 248 (2014), 225-234.
10. H. Zegeye (2015): Convergence theorems for Bregman strongly nonexpansive mappings in reflexive Banach spaces, *Filomat,* 28:7 (2014),1525-1536.
11. H. Zegeye and O. A. Daman (2015): Approximating the minimum-norm fixed point of pseudocontractive mappings, Asian-European Journal of Mathematics, Vol. 8, No. 2 (2015) 1550036 (13 pages), DOI: 10.1142/S1793557115500369.

1.2.2 Algebra and Geometry Prof. J-B Gatsinzi & Prof. Massamba: In mathematical systems theory, the plant of a deterministic dynamical system may be defined by a map, which has an input space as its domain and an output space as its range. Traditionally, these systems have been modelled by differential equations in Euclidean spaces. However, a significant advancement was made when the great mathematician Sophus Lie created a revolutionary discipline in mathematics called Lie Group Theory. The motivation behind Lie's work was to find a structured approach for the advance treatment of differential equations. Lie groups have an algebraic structure and are also subsets of space, which implies that they have geometry. Some of these have properties similar to Euclidean space, making it possible to do analysis on them. In differential geometry, one considers abstract spaces called manifolds, which resemble Euclidean space locally but have a very general global structure. An important feature of manifolds is that they are equipped with a measure of distances. Recent advances in differential geometry include an elegant geometric interpretation of Lie groups and their associated algebras. A contemporary approach to dynamical systems involves the geometric interpretation of the evolution of differential equations on manifolds and the study of

corresponding groups. Thus group theory and differential geometry provide a natural framework for the study of deterministic dynamical systems.

This group of Algebra and Geometry plans to use modern techniques of algebraic topology, and relates them to several other areas of mathematics such as operads, PROPs, field theories, and Gerstenhaber and Batalin-Vilkovisky algebras, moduli spaces of Riemann and symplectic Geometry which is connected to Morse Theory and Floer homology.

The team of mathematicians in this area consists of:
1. Prof. Jean-Baptiste Gatsinzi /coordinator/, University of Botswana
2. Prof. Themba Dube/ University of South Africa, South Africa
3. Prof. Fortuné Massamba/ University of KwazuluNatal, South Africa
4. Dr. Martin Mogochi/University of Namibia, Namibia
5. Dr. Kwashira Rugare /University of Botswana, Botswana

Publications in the area of Algebra and Geometry related to the RGSMA project:

1. **J.–B Gatsinzi** (2013). Brackets in the Free Loop Space Homology of Some Homogeneous Spaces. African Diaspora Journal of Mathematics. Volume 16, 28-36.
2. **J.–B Gatsinzi** (2013). Loop Space Homology of Some Homogeneous Spaces. International Science and Technology Journal of Namibia, Volume 1, 47-54.
3. **J.–B Gatsinzi** (2014). A Model for Function Spaces. Topology and its Applications. Volume 168, 153-158.
4. **F. Massamba**(2013). Symmetries of Null Geometry in Indefinite Kenmotsu Manifolds. Mediterranean Journal of Mathematics. Volume 10, Number 2, 1079-1100.
5. **F. Massamba** (2013). LighlikeHypersurfaces in indefinite Trans-Sasakian Manifolds. Results in Mathematics. Volume 63, Number 1, 251-287.
6. **F. Massamba and T. Tshituna-Matamba.** Horizontally Submersions of Contact CR-Submanifolds. Turkish Journal of Mathematics (Accepted).
7. **F. Massamba and T. Tshituna-Matamba:** Horizontally submersions of contact CR-submanifolds, Turkish Journal of Mathematics, 38 (2014), no. 3. 436-453
8. **A. Banyaga and F. Massamba**. Non-existence of Certain Einstein Metrics on Some Symmetric Manifolds (Submitted).
9. **A. Banyaga and F. Massamba:** Non-existence of certain Einstein metrics on somes symmetric manifolds, Forum Mathematicum, *In press*
10. **F. Massamba:** Almost Weyl structures on null geometry in indefinite Kenmotsu manifolds, Mathematica Slovaca, *In press*
11. -J.B. Gatsinzi, Euclidean Geometry (book in press)
12. - R. Kwashira and J.-B Gatsinzi, Korean Journal of Mathematics (2015, to appear)

The following table shows the number of students assisted by Simon's Foundation Fund

	2013/14 Academic Year		2014/15 Acad. Year		20115/16 Acad. Year	
	PhD	MSc	PhD	MSc	PhD	MSc
Male	9	8	10	8	10	8
Female	3	2	2	3	3	1
Total	12	10	12	11	13	9

Remarks: So far one PhD and two Msc have completed their studies with the support of Simon's Foundation.